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Terminal Spacecraft Coplanar Rendezvous Control

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The dynamics of relative motion between two nearby spacecraft for rendezvous is investigated in a local orbital coordinate system. An analysis by the phase plane method shows that a stable equilibrium state may exist in the motion. Based on this analysis, a control method called the range-rate control algorithm (RRCA) has been established. In addition, an omnidirectional version of the RRCA has also been introduced. The distance as well as the distance rate between the two spacecraft decrease exponentially to zero. The controlled trajectory is a stable and straight line in the orbital plane, whose orientation can be located freely. The numerical computation, correlated measurement, and propulsive implementation for the control algorithm are simple. A spacecraft rendezvous simulation is given as illustrative example.

Introduction

THE rendezvous operation of a maneuvering spacecraft to a target spacecraft usually consists of two successive phases: long-distance navigation and short-distance homing. In the navigation phase, the orbit parameters as well as the distance between the two spacecraft may be seriously diverged. The main control task in this phase is impulsive orbit transfer, which requires both orbit determination and impulse generation techniques. This is a kind of open-loop control. The control accuracy achievable by these techniques is rather below that required for the subsequent docking of the two spacecraft. Therefore, what to do in the navigation phase is to bring the maneuvering spacecraft into the vicinity (say, <100 km) in the same orbital plane of the target spacecraft, where the homing phase (terminal rendezvous control) begins.

In the homing phase, the relative motion is approximated by a system of linear differential equations for which an analytical solution exists.[1] The solution depends on the initial state of the motion and, in particular, on the relative velocity between the two spacecraft at a given initial moment; one can then influence the future course of the motion by adding impulsive velocity to the maneuvering spacecraft at this moment. The control method based on analytical solution is regarded as a quantitative method. There are various proposals on how to apply the impulses, but the essence of the problem remains the same as for the navigation phase, that is, open-loop control with inadequate accuracy of the state measurement and impulse generation. Necessarily, there must be a closed-loop terminal rendezvous control for a precise and safe spacecraft docking. Besides, the system of equations of motion is nonlinear in general, to which the analytical solution seldom exists. Therefore, the quantitative method is restricted in application.

One kind of closed-loop control is the widely used proportional navigation law,[2] which removes the component of the relative velocity transverse to the line of sight between the two spacecraft. This control originates in kinematics and does not account for the specific dynamics of in-orbit rendezvous motion. Therefore, it presents little insight into the problem.

This paper presents some main results of the author's past work and some new ideas of the problem.

Assume the target spacecraft is in a circle orbit with orbital angular velocity ω. The orbital coordinate system $Txyz$ of target spacecraft is shown in Fig. 1. The axis Ty is directed upward away from Earth's center; the axis Tz aligns with the normal to the orbital plane and the axis Tx completes the right-hand system. The position of the maneuvering spacecraft in the system is given by a vector radius $\bar{\rho}$. Then, the coplanar relative motion between the two spacecraft is determined by $\bar{\rho}$ and its evolution in time, which is described by the following equations in linearized gravitation terms:

$$\rho'' - \rho(\omega + \varphi')^2 - \rho\omega^2(3\sin^2\varphi - 1) = a_\rho \tag{1}$$

$$\rho\varphi'' + 2\rho'(\omega + \varphi') - 1.5\rho\omega^2 \sin 2\varphi = a_\varphi \tag{2}$$

Equation (1) describes the variation of the magnitude of $\bar{\rho}$, which is called the distance motion (DM); Eq. (2) describes the variation of the direction of $\bar{\rho}$, which is called the phase angle motion (PM). The quantities a_ρ and a_φ are the differences between the components of the accelerations of the two spacecraft, resulting from all applied and external forces (except for gravitation) such as propulsive and other disturbance forces.

Instead of a quantitative method, the qualitative method adopted in this paper will program the control variables in the system and analyze the resulted system's dynamics such as equilibrium states in the motion, stability analysis of the equilibrium states, and control law synthesis of the motion. The importance of dynamics analysis is obvious. The elaborated gravitational stabilization theory of satellite attitude motion is a good example. Owing to this theory, there exists a stable equilibrium state in which one of the satellite body axes aligns

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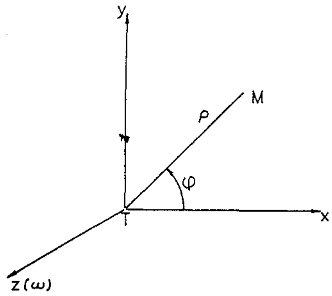

Fig. 1 Reference coordinate system.

with the orbital radius, that is, a simple three-axis attitude control for the Earth-pointed satellite can be implemented. Now, by analogy, a question may well be asked: Is there any stable equilibrium state as well in the relative motion between the two spacecraft? If there is one, then what control law is required in this case?

Equilibria, Stability, and Control

A special control program is expected if the controlled relative trajectory is a straight line, i.e., $\varphi = \varphi_s = \text{const}$, that Eq. (2) reduces to (assuming $a_\varphi = 0$)

$$\rho' = \tfrac{1}{4}(3 \sin 2\varphi_s)\omega\rho$$

or in general

$$\rho' = k\omega\rho$$

Then, a control program for DM can be suggested as

$$a_\rho = c(\rho'_{\text{pr}} - \rho'), \qquad c > 0 \tag{3}$$

$$\rho'_{\text{pr}} = k\omega\rho \tag{4}$$

Figure 2 shows the control error $e = \rho' - \rho'_{\text{pr}}$ as well as $r = e/|\rho'|$ in the rendezvous process. Because the error is so small, the controlled DM can be approximated by Eq. (4): $\rho' \approx \rho'_{\text{pr}} = k\omega\rho$.

Parameter k is choosable and must be negative for spacecraft rendezvous. When k is selected, the DM is approximately determined by Eq. (4). To analyze PM, one substitutes Eq. (4) into Eq. (2) and obtains the following equation for PM (assume $a_\varphi = 0$):

$$\varphi'' + 2k\omega\varphi' - 1.5\omega^2 \sin 2\varphi = -2k\omega^2 \tag{5}$$

The stationary solution of Eq. (5) is defined as equilibrium state φ_e determined by

$$E(\varphi) = \sin 2\varphi_e - \tfrac{4}{3}k = 0 \tag{6}$$

The condition for equilibrium state to exist is $k \in (-0.75, 0)$. The -0.75 is called a critical value of k, below which no equilibrium state can exist. For each selected k, there are four equilibrium states (Fig. 3): φ_{e1}, φ_{e2}, φ_{e3}, φ_{e4}. Here, φ_{e1} and φ_{e3} correspond to the negative slope of $E(\varphi)$ ($\cos 2\varphi < 0$), and they are 180 deg apart; φ_{e2} and φ_{e4} correspond to the positive slope of $E(\varphi)$ ($\cos 2\varphi > 0$), and they are also 180 deg apart.

In the equilibrium states, the maneuvering spacecraft is moving to the target spacecraft along a straight line in the orbital frame of reference, whose orientation is given by φ_e. The range and range rate between the two spacecraft are decreasing and ideally are determined by Eq. (4): $\rho = \rho_0 e^{k\omega t}$, $\rho' = \rho'_0 e^{k\omega t}$. The terms ρ_0 and ρ'_0 are the initial values of the motion in the equilibrium state.

On the one hand, this control program is simple. It requires only to measure the range and range rate. It adjusts only in-line propulsion a_ρ. On the other hand, this control is very attractive because the controlled trajectory is certainly a straight line of known orientation in the reference system. However, whether or not this control really works depends on the stability of the equilibrium state. Only a stable equilibrium state enables the control to work in practice.

To investigate the stability of equilibrium states, the well-known technique of linearized motion of small deviation is adopted. Thus,

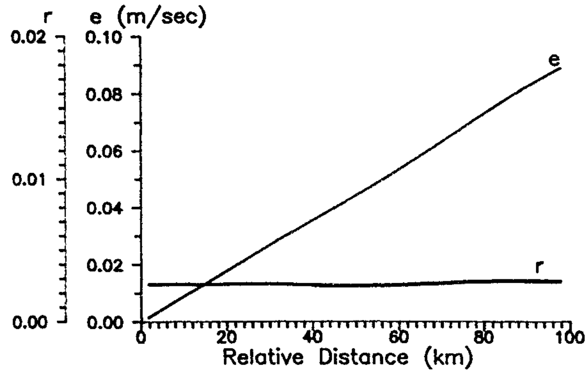

Fig. 2 Control error of DM.

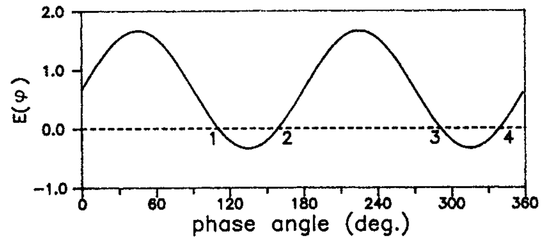

Fig. 3 Location of equilibrium states.

the PM is expanded about the equilibrium states to obtain so-called linearized motion as follows:

$$\begin{aligned} \tilde{\varphi}'' + 2k\omega\tilde{\varphi}' - 3\omega^2 \cos 2\varphi_e \tilde{\varphi} &= 0 \\ \tilde{\varphi} &= \varphi - \varphi_e \end{aligned} \tag{7}$$

The eigenvalues of Eq. (7) are

$$\lambda_{1,2} = -k\omega \pm \omega\sqrt{k^2 + 3\cos 2\varphi_e}$$

An equilibrium state is stable if all its eigenvalues have negative real parts. Because $k < 0$ none of the four equilibrium states is stable. From the mechanics point of view, the equilibria represent a dynamic balance between the gravitational, centrifugal, and Coriolis forces in the direction of the PM. The balance is unstable in general. To enhance the stability, one or more of these forces should be varied. The Coriolis force, which is proportional to ρ', is the only possible force to control. Therefore, the DM has to be reprogrammed and Eq. (4) is modified by including the dynamic variable φ' as follows:

$$\rho'_{\text{pr}} = \frac{k\omega + k_1\varphi'}{\omega + \varphi'}\omega\rho, \qquad k_1 > 0 \tag{8}$$

Equation (8) has been defined as the range-rate control algorithm (RRCA).[3,4] According to the RRCA, the PM is rewritten as

$$\varphi'' + 2k_1\omega\varphi' - 1.5\omega^2 \sin 2\varphi = -2k\omega^2 \tag{9}$$

The RRCA has the same equilibrium states shown in Fig. 3. But their stability now is totally different. The linearized motion of Eq. (9) has the following eigenvalues:

$$\lambda_{1,2} = -k_1\omega \pm \omega\sqrt{k_1^2 + 3\cos 2\varphi_e}$$

Because $k_1 > 0$, the equilibrium states φ_{e1} and φ_{e3} (for which $\cos 2\varphi_e < 0$ by the above definition) are stable. In the phase plane (φ, φ'), φ_{e1} and φ_{e3} might be stable nodes or foci depending on whether or not $k_1^2 + 3\cos 2\varphi_e > 0$; φ_{e2} and φ_{e4} (for which $\cos 2\varphi_e > 0$ by definition) are unstable equilibrium states (saddle points). They separate the stable equilibrium states φ_{e1} and φ_{e3} in the plane (φ, φ').

What has been established so far is the local stability of the equilibrium states, that is, the stability of PM near the equilibrium states. The PM far from the equilibrium states is the subject of global stability analysis.

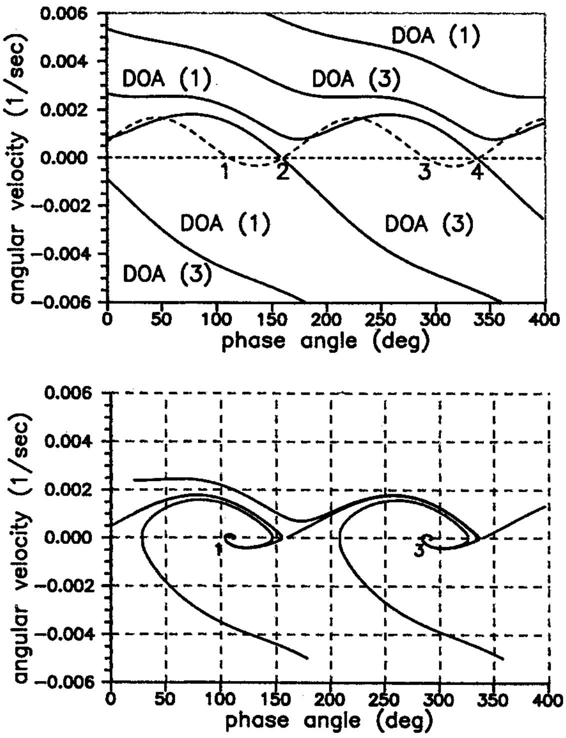

Fig. 4 Domains of attraction/phase plane trajectories (foci: $k = -0.5$, $k_1 = 0.5$).

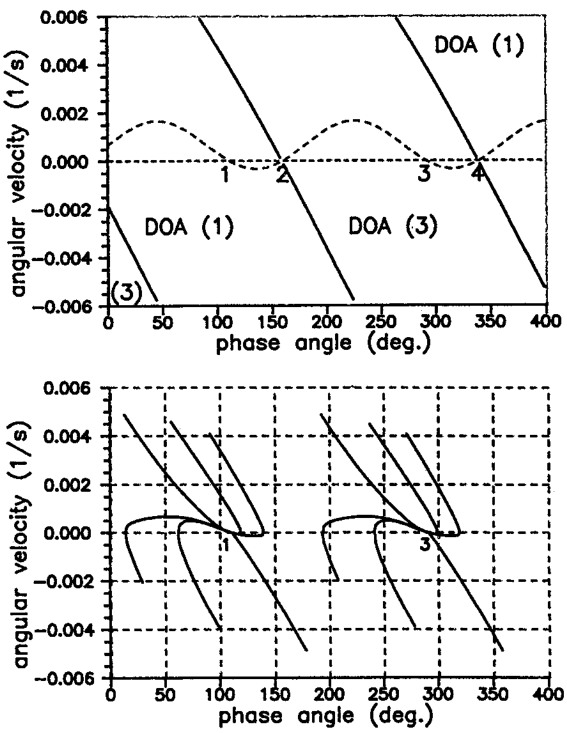

Fig. 5 Domains of attraction/phase plane trajectories (node: $k = -0.5$, $k_1 = 2.0$).

One method of global stability analysis is to build a domain of attraction (DOA) of the stable equilibrium states in the phase plane (φ, φ').[4,5] Figures 4 and 5 give some variants of the DOA and the associated phase plane trajectories. The form and size of the DOA depend on the values of k, k_1, and ω. If the initial state of a PM is inside the DOA of one of the stable equilibrium states, the PM will ultimately converge to the stable equilibrium state after some transient process. Two such trajectories (in the upper and lower plane) as well as the in-line propulsion's variation are shown in Figs. 6 and 7.

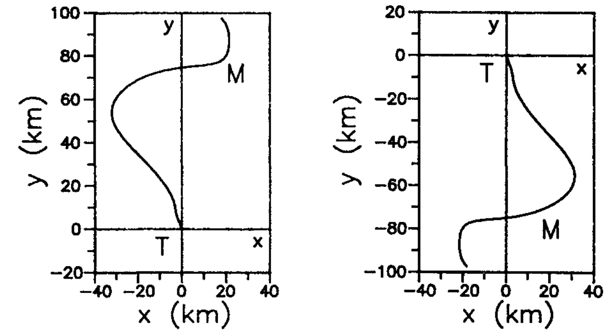

Fig. 6 In-plane rendezvous trajectory by RRCA ($k = -0.5, k_1 = 0.5$).

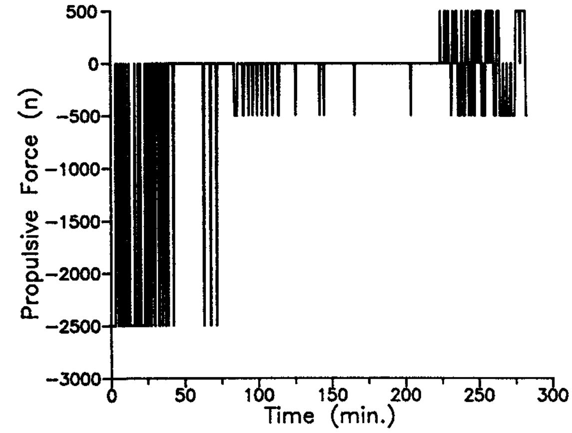

Fig. 7 In-line propulsion's variation (sampling time 25 s).

Omnidirectional RRCA

Because the parameter k is limited to $(-0.75, 0)$ in the RRCA, the stable and straight-line trajectories of the equilibrium states can exist only in a specific sector of the plane Txy (see Fig. 6). The sector is half of the second quadrant by the axis Ty if the maneuvering spacecraft is approaching the target spacecraft in the upper plane ($y > 0$) and half of the fourth quadrant by the axis Ty if the approach is in the lower plane. Besides, the range rate ρ' cannot be large, a fact that makes the controlled process relatively long. To overcome these problems, a propulsion control a_φ of the PM is turned on in addition to the in-line propulsion control a_ρ. The propulsive force a_φ may dramatically affect the dynamic balance of the PM mentioned above; that is, the stable and straight-line trajectories can be located anywhere in the whole orbital plane (omnidirectionally), and the parameter k can be chosen in principle as large as needed. This is the so-called omnidirectional range-rate control algorithm (ODRRCA).[6] The ODRRCA consists of two parts: 1) in-line propulsion control based on the RRCA of Eq. (8) and 2) the PM's propulsion, controlled according to the law

$$a_\varphi = \rho\omega^2(a \sin 2\varphi + b \cos 2\varphi) \tag{10}$$

The coefficients a, b of Eq. (10) are determined such that the equilibrium states of the ODRRCA meet the requirements of stability and omnidirectionality. Substitute Eqs. (8) and (10) into Eq. (2) to obtain the PM as

$$\varphi'' + 2k_1\omega\varphi' - (a + 1.5)\omega^2 \sin 2\varphi - b\omega^2 \cos 2\varphi = -2k\omega^2 \tag{11}$$

The equilibrium state φ_{od} of Eq. (11) satisfies the relation

$$(a + 1.5)\sin 2\varphi_{\mathrm{od}} + b \cos 2\varphi_{\mathrm{od}} = 2k \tag{12}$$

The linearized motion of Eq. (11) is given by

$$\tilde{\varphi}'' + 2k_1\omega\tilde{\varphi}' + q\omega^2\tilde{\varphi} = 0, \qquad \tilde{\varphi} = \varphi - \varphi_{\mathrm{od}} \tag{13}$$

$$q = -2(a + 1.5)\cos 2\varphi_{\mathrm{od}} + 2b \sin 2\varphi_{\mathrm{od}} \tag{14}$$

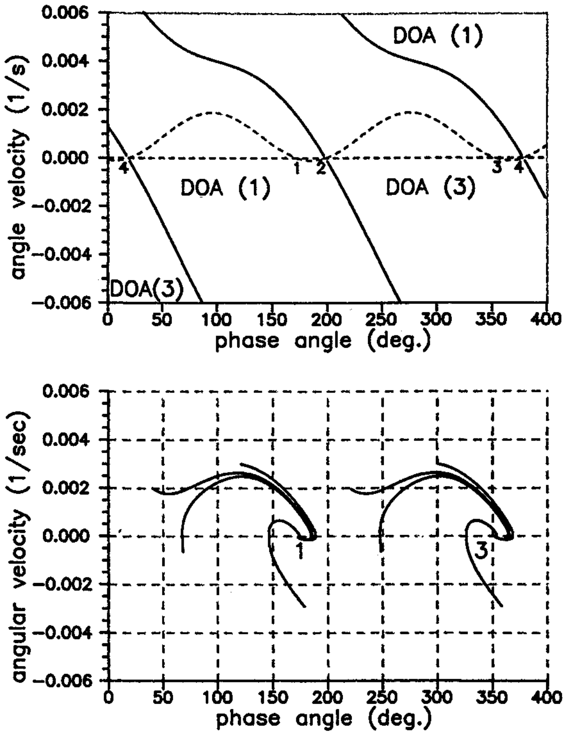

Fig. 8 Domains of attraction/phase plane trajectories ($k = -2.0$, $k_1 = 1.5$, $q = 4.0$, $\varphi_{od} = 172$ deg).

The parameter q must be given positive because of the stability requirement [see Eq. (13)]. The parameter φ_{od} can be fixed at any desired value from 0 deg to 360 deg to meet the omnidirectionality requirement.

Coefficients a, b are then solved from Eqs. (12) and (14) as function of φ_{od} and q:

$$a = -1.5 + 2k \sin 2\varphi_{od} - 0.5q \cos 2\varphi_{od} \tag{15}$$

$$b = 2k \cos 2\varphi_{od} + 0.5q \sin 2\varphi_{od} \tag{16}$$

Such a, b will make the controlled trajectories both stable and omnidirectional, Substitute Eqs. (15) and (16) into Eq. (11) to obtain a new form of PM:

$$\varphi'' + 2k_1\omega\varphi' + c\omega^2 \cos(2\varphi - 2\varphi_{od} - \alpha) = c\omega^2 \cos\alpha \tag{17}$$

$$c = \sqrt{4k^2 + 0.25q^2}, \qquad \sin\alpha = \frac{q}{2c}, \qquad \cos\alpha = -\frac{2k}{c}$$

It is easy to verify that two equilibrium states of Eq. (17) are $\varphi_{e1} = \varphi_{od}$ and $\varphi_{e3} = \varphi_{od} + 180$ deg. They are stable because $q > 0$. They are also omnidirectional because of the free choice of $\varphi_{od} \in [0$ deg, 360 deg]. The other two equilibrium states $\varphi_{e2} = \varphi_{od} + \alpha$ and $\varphi_{e4} = \varphi_{od} + \alpha + 180$ deg are unstable; they separate φ_{e1} and φ_{e3} in the phase plane.

In principle, k is not confined in the ODRRCA. However, a reasonable value of k (e.g., $k = -2.0$) must be selected to speed up the rendezvous process and at the same time to ensure a suitable DOA for the stable equilibrium states of the motion (Fig. 8). The parameters of the ODRRCA such as k, k_1, φ_{od}, q may also be considered as

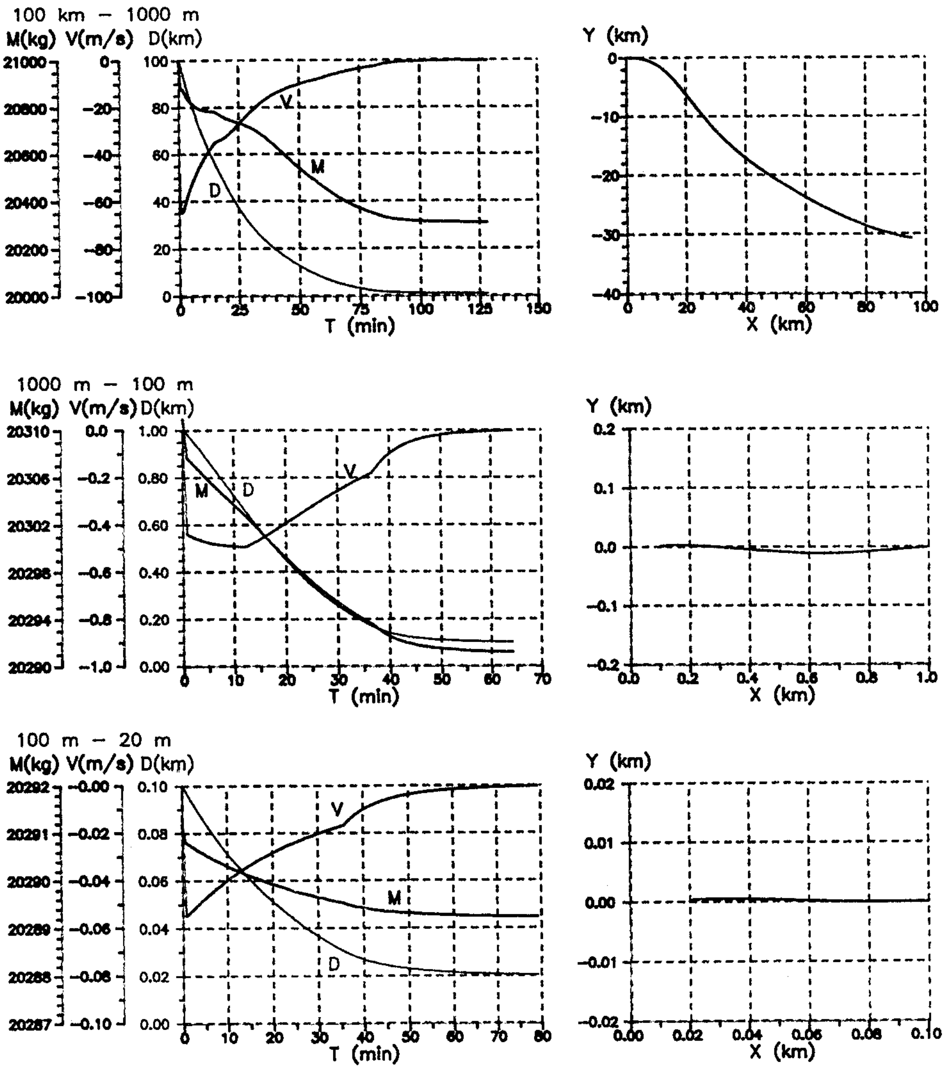

Fig. 9 Simulation of a space plane rendezvous by ODRRCA.

control variables during the process so that some optimal controls may be determined.[6]

Computer Simulation

A computer simulation of the European HERMES spaceplane rendezvous with a space station has been conducted. The terminal rendezvous is divided into three successive subprocesses in distance: 100 km–1 km, 1 km–100 m, and 100 m–20 m.[7] By the end of each subprocess the space plane should be at one of the hold points located on the horizontal at 1 km, 100 m, and 20 m behind the space station. Figure 9 shows the variations of the distance D, distance rate V, and mass M of the space plane; the in-plane trajectories are also given for each subprocess.

Conclusions

The RRCA and its modification the ODRRCA are given to control the spacecraft rendezvous in low Earth orbit. What is important is that the algorithm ensures the existence of a stable equilibrium state in the motion. The controlled trajectory is stationary, stable, and straight line. The orientation of the straight-line trajectory can be located completely free for the ODRRCA and partially free for the RRCA. This is a useful technique for spacecraft rendezvous and docking. It can also be applied to control the departing and stationkeeping motion between two spacecraft. For departure k must be positive; for stationkeeping $k = 0$. The realization of the control, that is, the computation, measurement, and propulsion implementation, for the algorithm is very simple.

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